Name:___________________________ Date:_________________________

SOLUTION: HW Pre Calculus 12: Section 1.2 Horizontal and Vertical Translations

1. Suppose you are given a function $f(x)=3(x-4)^2+5$ and you replace the variable "x" in the equation with $x+3$, then answer the following:

i) What does the equation become after the transformation?

When we replace "x" with "x+3", the equation will become:

$f(x)=3((x+3)-4)^2+5$ and then simplify the equation:

$f(x)=3(x-1)^2+5$

ii) Which way if the graph shifted and by how much?

The graph will be shifted 3 units to the left

iii) Where is the vertex of the parabola after the transformation?

The original vertex of the parabola is (4,5). If we shift the parabola 3 units to the left, all the x-coordinates will subtract 3. So the new vertex will be (1,5)

2. Suppose you are given a function $f(x)=2x^3-8x^2+5$ and you replace the variable "x" in the equation with $x-2$ and the "y" variable with $y+10$, then answer the following:

i) What does the equation become after the transformations? Simplify your equation & isolate the "Y" variable.

$(y+10)=2(x-2)^3-8(x-2)^2+5$.

The "y" variable becomes "y+10" and the "x" variable becomes "x-2". Now simplify the equation:

$y=2(x^3-2x^2+4x-8)-8(x^2-4x+4)-5$

$y=2x^3-4x^2+4x-16-8x^2+32x-32-5$

$y=2x^3-12x^2+36x-53$

ii) Which way if the graph shifted and by how much?

The graph is shifted 10 units DOWN and 2 units to the RIGHT

3. Indicate the transformation from the function on the left to the function on the right:

a) $y=x^2+3x+4 \rightarrow y=(x-5)^2+3(x-5)+4$

The "x" variable is replaced with "x-5", this will shift the function 5 units to the RIGHT!!

b) $y = x^2 + 4x + 5 \to y = x^2 + 4x + 11$

The left side of the equation is increased by 6. If we move this value to the left, it looks like this:

$y = x^2 + 4x + 5 \to y - 6 = x^2 + 4x + 5$

So our transformation is "y" is transformed to "y-6", this will shift the function 6 units UP!!

c) $y = \sqrt{3x-4} \to y = \sqrt{3x-4} + 10$

The value of 10 is added "OUTSIDE" of the function, therefore, it is a vertical displacement. Again, move the 10 to the left and we have:

$y = \sqrt{3x-4} \to y - 10 = \sqrt{3x-4}$. The function is shifted 10 units UP!!

d) $y = \sqrt{3x-4} \to y = \sqrt{3x-7}$

The value of 7 is increased inside the function, so that means it is a horizontal displacement. However, we can't have a transformation like this: $3x-4 \to 3x-7$, you can't do this b/c there's a coefficient with the "x". Therefore, if there's a coefficient, FACTOR it OUT FIRST!!!!

$y = \sqrt{3\left(x - \frac{4}{3}\right)} \to y = \sqrt{3\left(x - \frac{7}{3}\right)} \to y = \sqrt{3\left(x - \frac{3}{3} - \frac{4}{3}\right)}$. From here we can see that "x" is transformed to "x - 3/3", which is "x-1". Therefore, the function is shifted 1 unit to the RIGHT!!

e) $y = |x| \to y = |x-2| + 4$

"x" → "x – 2" : Which is a Horizontal shift of 2 units to the RIGHT
"y" → "y – 4" : Which is a vertical shift of 4 units UP

f) $y = \sqrt{x} \to y = \sqrt{x+3} - 7$

"x" → "x+3" : Horizontal SHIFT 3 UNITS LEFT!!
"y" → "y+7" : Vertical SHIFT of 7 UNITS DOWN!!

g) $y = 3x + 2 \to y = 3x + 8$

This one can be perceived as either a Horizontal Shift OOORRR a VERTICAL shift. The vertical is EASIER….

$y - 2 = 3x \to y - 8 = 3x$: This makes it a Vertical shift of 6 unit UP!!

$y = 3(x+k) + 2 \to y = 3x + 8$

$3x + 3k + 2 = 3x + 8$

$3k + 2 = 8$: This is a horizontal shift of 2 units to the LEFT!!

$3k = 6$

$k = 2$

In general for lines, there could be infinite answers because any point on a line can align onto any other point on a linear function. Just as long as the lines overlap, your answer can be anything. However, this only works for linear functions!!!

h) $y = x^2 \to y = x^2 + 6x + 12$

For this question, complete the square for the right side. If you don't know how to complete the square, go review your PC11 notes.

$$y = x^2 \to y = x^2 + 6x + 12$$
$$y = x^2 + 6x + 9 - 9 + 12$$
$$y = \left(x^2 + 6x + 9\right) - 9 + 12$$
$$y = \left(x + 3\right)^2 + 3$$

So we have: "x" → "x+3": 3 units LEFT and "y" → "y-3" : 3 units UP!!

i) $y = \frac{1}{x} \to y = \frac{1}{x+5} + 3$

"x" → "x+5" : 5 units LEFT
"y" → "y-3" : 3 units UP!!

4. Given each equation for $y = f\left(x\right)$, indicate the new equation after each translation:

a. $f\left(x\right) = 3x - 5$ A horizontal shift of 3 units right and 2 units up

For these types of questions, write the transformations out first!!

ie: x → "x-3" so our equation becomes: $y = 3\left(x - 3\right) - 5$

Y → "y -2": the equation is now: $y - 2 = 3\left(x - 3\right) - 5$

now take the time to simplify your eqation:
$$y - 2 = 3x - 9 - 5$$
$$y = 3x - 12$$

b. $f\left(x\right) = 2x^2 + 3$ A horizontal shift of 5 units left and 8 units up

First transformation: "x" → "x + 5' $f\left(x\right) = 2\left(x + 5\right)^2 + 3$

2nd transformation : « y » → ' y – 8 '
$$y - 8 = 2\left(x + 5\right)^2 + 3$$
$$y = 2\left(x + 5\right)^2 + 11$$

c. $f\left(x\right) = \sqrt{x - 5} + 1$ A horizontal shift of 7 units right and 6 units down

$$f(x)+6=\sqrt{(x-7)-5}+1$$
$$y=\sqrt{x-12}-5$$

d. $f(x)=3^x+2$ A horizontal shift of 11 units left and 2 units down

$$f(x)+2=3^{x+11}+2$$
$$y=3^{x+11}$$

e. $x^2+y^2=16$ A horizontal shift of 4 units right and 8 units up

$$x^2+y^2=16$$
$$(x-4)^2+(y-8)^2=16$$

f) $f(x)=\dfrac{1}{x-3}$ A horizontal shift of 2 units left and 6 units down

$$y+6=\frac{1}{(x+2)-3}$$
$$y=\frac{1}{x-1}-6$$

g) $f(x)=x^3+2x^2$ A horizontal shift of 5 units right and 11 units down

$y+11=(x-5)^3+2(x-5)^2$. Take the time to expand this….

5. Given the graph of $y=f(x)$, draw the resulting image after each transformation:

When graphing transformations, take several key points on the original graph. For instance, in the first image, the key points are (-6, -6) , (-2, 4) , and (5, 4).

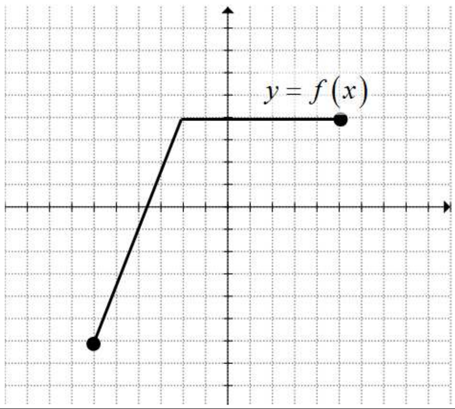
Next we indicate the shifts involved.

i) 3 units to the right and 1 unit UP! All the "x" coordinates will increase by 3 and all the "y" coordinates will increase by 1.

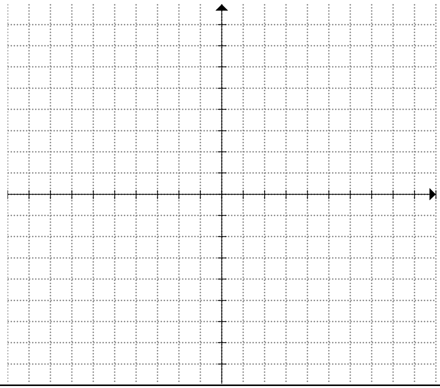
So that means the new points will be: (-3, -5) , (1, 5) and (8, 5).

ii) The second function shifts the graph 3 units UP and 1 unit LEFT. The "X" coordinates will decrease by 1 and the "Y" coordinates will increase by 3.

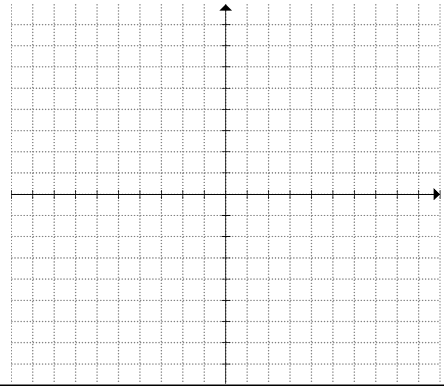
The new points will be: (-7, -3) , (-3, 7) , and (4, 7).

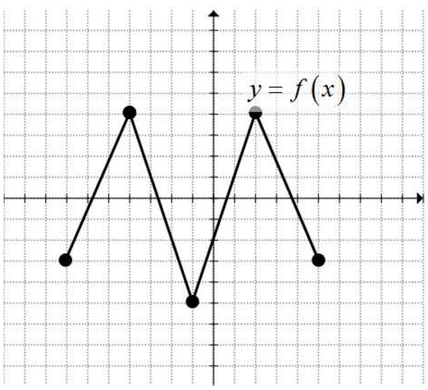


i) $y = f(x-3)+1$

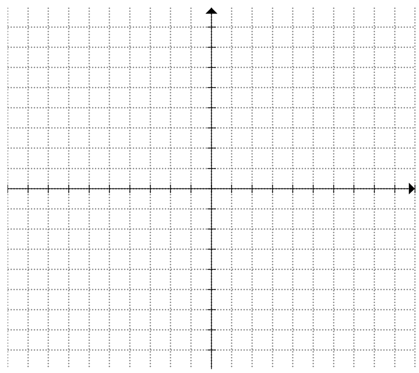


ii) $y-3 = f(x+1)$

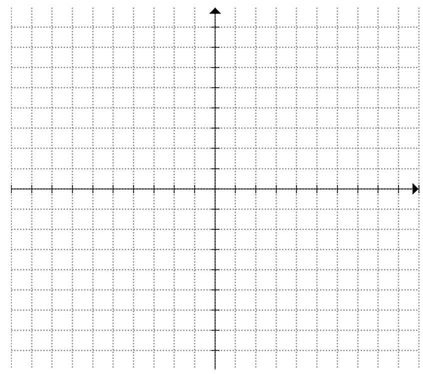




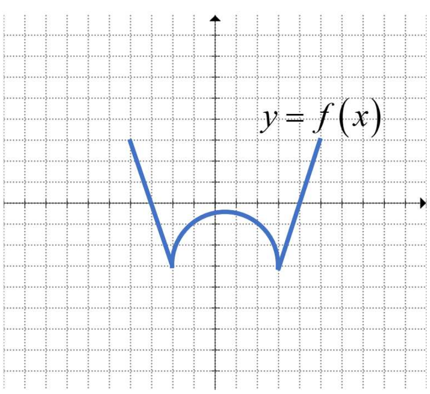
i) $y+1 = f(x+2)$



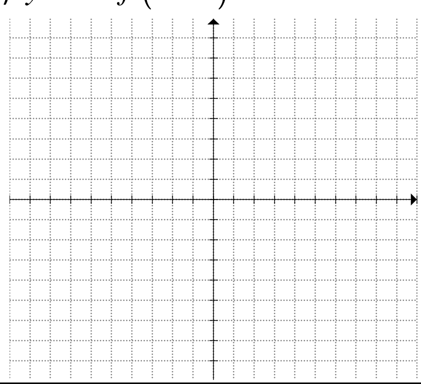
ii) $y = 1+f(x+4)-2$



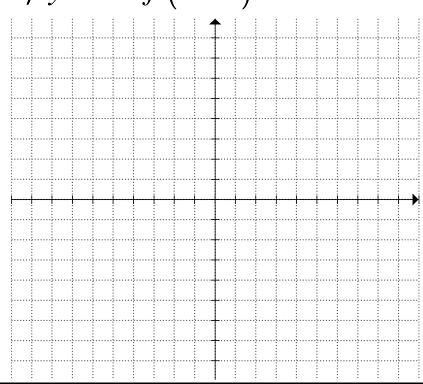
Again, the first step should selecting the key points on the graph. We have: (-7, -3) , (-4, 4) , (-1, -5) , (2,4) and (5,-3).
I) The first transformation is 1 unit DOWN and 2 units LEFT. So all the "x" coordinates will decrease by 2 and all the "Y" coordinates will decrease by 1. The new coordinates will be: (-9, -4) (-6, 3), (-3, -6), (0, 3) and (3, -4). Graph these points.....
ii) The second transformation is: 1 units DOWN and 4 units left. "X" coordinates subtract 1 and "y" coordinates subtract 4. The new points are: (-8, -7) , (-5, 0) , (-2, -9) , (1, 0) (4, -7).



i) $y-3 = f(x-4)$



ii) $y-2 = f(x+3)-5$



6. Given that the coordinates of (2,3) is transformed to (8,4) from $y = f(x) \rightarrow y = f(x-a)+b$, what is the value of a + b?

To go from (2,3) to (8,4) we shift 6 units RIGHT and 1 units UP.
The value of "a" will be 6 and the value of "b" will be 1.
a+b = 7

7. Given that the coordinates (a,b) are on the function $y = f(x)$, find the new coordinates for each function after the transformation:

i) $y = f(x-3)+2$	ii) $y-5 = f(x+1)+2$
iii) $y = f(x+7)-11$	iv) $y-4 = f(x-5)+3$

8. Suppose the point (a,b) is on the function from the left. What will the point become after the transformation shown from the function on the right? Indicate all possible answers:

a) $y = |3x-2| \rightarrow f(x) = |3x+12|+4$

When there is a coefficient in front of the "x", factor it out first.
YOU can't transform 3x-2 to 3x+12 directly. Instead, factor and then deal with the transformation

$$y = \left|3\left(x-\frac{2}{3}\right)\right| \rightarrow f(x) = |3(x+4)|+4$$

$$y = \left|3\left(x-\frac{2}{3}\right)\right| \rightarrow f(x) = \left|3\left(x+\frac{14}{3}-\frac{2}{3}\right)\right|+4$$

$$x \rightarrow x+\frac{14}{3}$$

$$y \rightarrow y-4$$

2nd Method

$$3(x+k)-2 = 3x+12$$

$$3x+3k-2 = 3x+12$$

$$3k-2 = 12$$

$$3k = 14$$

$$k = \frac{14}{3}$$ 14/3 units to the left!!

14/3 units to the right and 4 units UP!!

b) $y = \frac{4}{3}x+11 \rightarrow f(x) = \frac{4x+10}{3}$

THERE ARE TWO WAYS TO DO THIS QUESTION!!!

First method: treat it as a vertical displacement

For this question, rewrite the right side so that you split the two terms up.

$$y=\frac{4}{3}x+11\rightarrow y=\frac{4x}{3}+\frac{10}{3}$$

$$\rightarrow y=\frac{4x}{3}+11+\frac{10}{3}-11$$

$$\rightarrow y=\frac{4x}{3}+11-\frac{23}{3}$$

So the transformation is vertical. "y" → "y+23/3". The graph is shifted 23/3 units DOWN!!!

2nd method: Use $x\rightarrow x+k$, where you treat it as a horizontal translation

$$y=\frac{4}{3}(x+k)+11=\frac{4x}{3}+\frac{10}{3}$$

$$\frac{4x}{3}+\frac{4k}{3}+11=\frac{4x}{3}+\frac{10}{3}$$

$$\frac{4k}{3}+11=\frac{10}{3}$$

$$4k+33=10$$

$$4k=23$$

$$k=\frac{23}{4}\ units\ LEFT$$

c) $y=2^x+1\rightarrow f(x)=4(2^x)-3$

Turn every value into a power of 2, then combine exponents:

$$y=2^x+1\rightarrow y=2^2(2^x)-3$$

$$y=2^x+1\rightarrow y=2^{x+2}-3$$

If you rewrite it this way, you can see that the "x" variable is changed to x+2, which means it's a HS of 2 LEFT

d) $y=\frac{1}{x}\rightarrow f(x)=-1-x-x^2-x^3+\ldots\ \{0<x<1\}$

HINT: This is a geometric series - $a+ar+ar^2+ar^3+\ldots\ldots=\frac{a}{1-r}$ $\{0<r<1\}$

9. The parabola $y = x^2 - 4x + 3$ is translated 5 units right. In this new position, the equation of the parabola is $y = x^2 - 14x + d$. What is the value of "d"?

